The Second Moment of the Structure Function for Pseudoscalar Mesons in QCD Sum Rules

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Abstract

We calculate the values of the second moment M_2^s of the flavor-singlet structure function F_2^s for the pion and the kaon in QCD sum rules, and investigate how the results depend on their flavor structure (quark contents). Our calculations give similar values of M_2^s for these two mesons, because of cancellation among several non-small factors. We emphasize that decay constants, meson masses and quark masses play an essential role in the above cancellation.

Structure functions carry information on dynamical distributions of quarks and gluons inside hadrons. Intensive studies of nucleon structure functions have been performed using various lepton induced reactions. In contrast, those of other hadrons are not easy to access. It is not yet clear how the difference in quark contents of hadrons affects the structure functions. In the present paper, we would like to study this problem by calculating the values of the second moment M_2^s of the flavor-singlet structure function F_2^s for the pion and the kaon in QCD sum rules (QSR)

The QSR method, developed by Shifman, Vainshtein and Zakharov [1], has been used to study a number of problems in hadron physics [2]. Belyaev and Ioffe studied the Bjorken variable dependence of the structure function $F_2(\xi, Q^2)$ of the nucleon based on QSR, and showed that their method is applicable only in the intermediate region of the Bjorken variable ξ [3]. On the other hand, moments of structure functions can be studied by introducing constant external fields into QSR [4]. Using this technique, Belyaev and Blok calculated M_2^s values for the pion and the nucleon [5]. There also exist some applications of this technique to moments of other structure functions [6]. These calculations indicate that QSR is a useful tool in calculating moments of structure functions. Below we adopt this technique to investigate M_2^s values for the pion and the kaon, and study in particular the effects of decay constants, meson masses and quark masses on the momentum fraction carried by quarks.

The second moment $M_2^s(\mu^2)$ of the (unpolarized) flavor-singlet structure function F_2^s is defined as follows:

$$M_2^s(\mu^2) p_\mu p_\nu = \frac{1}{2} \langle \mathrm{hadron}(p) | \Theta^q_{\mu\nu} | \mathrm{hadron}(p) \rangle. \tag{1}$$

Here $|\mathrm{hadron}(p)\rangle$ is the momentum eigenstate of the hadron concerned, μ is the renormalization point of the operator, and $\Theta^q_{\mu\nu}$ denotes the quark part of the symmetrized energy-momentum tensor,

$$\Theta^q_{\mu\nu}(x) \equiv \frac{\mathrm{i}}{4} \left(\bar{\psi}(x) \overleftrightarrow{D}_\mu \gamma_\nu \psi(x) + \bar{\psi}(x) \overleftrightarrow{D}_\nu \gamma_\mu \psi(x) \right), \tag{2}$$

where $\psi(x)$ and D_μ are the quark field and the covariant derivative, respectively. M_2^s can also be represented in terms of (unpolarized) quark distribu-

tion functions $f_q(x,\mu^2)$:

$$M_2^s(\mu^2) = \int_0^1 \mathrm{d}\xi\ \xi\ \sum_q [f_q(\xi,\mu^2) + f_{\bar{q}}(\xi,\mu^2)]. \tag{3}$$

M_2^s can thus be interpreted as the momentum fraction carried by quarks.

In QSR, hadronic matrix elements of local operators, like M_2^s, can generally be calculated by introducing constant external fields [4]. In the present case, the external field has the tensor structure corresponding to the operator $\Theta^q_{\mu\nu}$.

$$\Delta\mathcal{L}(x) = -\Theta^q_{\rho\lambda}(x) S^{\rho\lambda}. \tag{4}$$

Under existence of this external tensor field, we consider the two-point function of the interpolating field for the hadron concerned. For pseudoscalar mesons, appropriate interpolating fields are axial vector currents.

$$j^5_\mu(x) = \left\{ \begin{array}{ll} \bar{u}(x)\gamma_\mu\gamma_5 d(x) & \text{for the pion,} \\ \bar{u}(x)\gamma_\mu\gamma_5 s(x) & \text{for the kaon,} \end{array} \right. \tag{5}$$

Thus the two-point function in the present case is given by

$$\Pi_{\mu\nu}(Q^2) = \mathrm{i} \int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}qx} \langle T[j^5_\mu(x), j^{5\dagger}_\nu(0)]\rangle_S \quad (Q^2 = -q^2), \tag{6}$$

where the subscript "S" represents existence of the external field. From Eq.(6), we extract the first order term $\Pi^1_{\mu\nu\rho\lambda}(Q^2)$ in the external field $S^{\rho\lambda}$ ($\Pi_{\mu\nu}(Q^2) = \Pi^0_{\mu\nu}(Q^2) + \Pi^1_{\mu\nu\rho\lambda}(Q^2)S^{\rho\lambda} + \ldots$).

According to the QSR method, we calculate $\Pi^1_{\mu\nu\rho\lambda}(Q^2)$ in two different ways. In the phenomenological side of QSR, we represent $\Pi^1_{\mu\nu\rho\lambda}(Q^2)$ in terms of M_2^s, using the reduction formula and the dispersion relation. On the other hand, in the operator product expansion (OPE) side of QSR, we expand $\Pi^1_{\mu\nu\rho\lambda}(Q^2)$ into terms with various condensates. In the region of Q^2 where these two calculations are manageable, we can equate both sides of QSR and can obtain QSR for M_2^s. In actual calculations we perform the Borel transformation on both sides of QSR.

First, we shall consider the phenomenological side of QSR. Hereafter we show calculations only for the kaon, since expressions for the pion are obtained in the same way. With the help of the reduction formula and the

double dispersion relation, we get

$$\begin{aligned}\Pi^1_{\mu\nu\rho\lambda}(Q^2) &= -4f_K^2\frac{M_2^s}{(Q^2+m_K^2)^2}q_\mu q_\nu q_\rho q_\lambda + \sum_i \frac{(B_i)_{\mu\nu\rho\lambda}}{(Q^2+m_K^2)(Q^2+m_i^2)} \\ &\quad + \sum_{ij}\frac{(C_{ij})_{\mu\nu\rho\lambda}}{(Q^2+m_i^2)(Q^2+m_j^2)} \\ &\quad + \Pi^1_{\mu\nu\rho\lambda}(Q^2 > s_0 : \text{perturbative}), \end{aligned} \tag{7}$$

where B_i and C_{ij} represent matrix elements which are not diagonal in the kaonic state, and m_i is the mass of the i-th excited state in the axial vector channel. And we assumed that perturbative calculations are correct in Q^2 above s_0.

In Eq.(7), M_2^s is appearing as the coefficient of $q_\mu q_\nu q_\rho q_\lambda$. So we extract the coefficient of $q_\mu q_\nu q_\rho q_\lambda$ from $\Pi^1_{\mu\nu\rho\lambda}(Q^2)$ and then apply the Borel transformation $\hat{L}_M$.

$$\hat{L}_M \equiv \lim_{\substack{Q^2, n\to\infty \\ \frac{Q^2}{n}=M^2}} \frac{1}{(n-1)!}(Q^2)^n\left(-\frac{\mathrm{d}}{\mathrm{d}Q^2}\right)^n, \tag{8}$$

where M is the Borel mass. In this way, we finally obtain

$$\hat{L}_M\Pi'^1(Q^2) = \left(-4f_K^2\frac{M_2^s}{M^4} + \frac{1}{M^2}b\right)\mathrm{e}^{-\frac{m_K^2}{M^2}} + \hat{L}_M\Pi'^1(Q^2 > s_0 : \text{perterbative}), \tag{9}$$

as the phenomenological side of QSR. Here b is a constant which comes from the second term of Eq.(7) and we neglected all exponentially small terms compared to the first term in the r.h.s. of Eq.(9).

On the other hand, in the OPE side of QSR we apply OPE to the T-product in the integrand of Eq.(6). To be consistent with the phenomenological side, we consider only the coefficient of $q_\mu q_\nu q_\rho q_\lambda$, and then perform the Borel transformation. After some calculations we obtain,

$$\begin{aligned}\hat{L}_M\Pi'^1(Q^2) &= -\frac{1}{2\pi^2}\frac{1}{M^2} - \frac{1}{18}\langle\alpha_s FF\rangle\frac{1}{M^6} - 2\frac{m_u\langle\bar{u}u\rangle + m_s\langle\bar{s}s\rangle}{M^6} \\ &\quad + \frac{1}{3}\frac{m_u\langle\bar{u}\sigma_{\mu\nu}F^{\mu\nu}u\rangle + m_s\langle\bar{s}\sigma_{\mu\nu}F^{\mu\nu}s\rangle}{M^8} \\ &\quad - \frac{16\pi}{27}\alpha_s\frac{\langle\bar{u}u\rangle^2 + \langle\bar{s}s\rangle^2}{M^8} + \frac{64\pi}{27}\alpha_s\frac{\langle\bar{u}u\rangle\langle\bar{s}s\rangle}{M^8}. \end{aligned} \tag{10}$$

Here we calculated up to dimension-6 condensate terms and the first order in quark masses, whereas in Ref.[5] they neglected quark masses. Wilson coefficients are obtained by calculating diagrams depicted in Fig.1. Note that the diagrams in which the external field interacts with vacuum quarks contribute to dimension-6 condensate terms, but they are small compared to the other dimension-6 condensate terms [5]. The smallness of this term is intuitively understandable, since low momentum quarks have less contribution to the momentum fraction. Fig.1

Equating Eq.(9) and Eq.(10), we can get QSR for M_2^s. But beforehand we have to perform the QCD evolution to get M_2^s for arbitrary renormalization point. After the QCD evolution, we get QSR for $M_2^s(\mu^2)$

$$\begin{aligned} & \frac{9}{25}(1-L^{50/81}) + L^{50/81}\frac{\mathrm{e}^{m_K^2/M^2}}{f_K^2}\left[\frac{1}{8\pi^2}M^2(1-\mathrm{e}^{-s_0/M^2})\right. \\ & + \frac{1}{72}\frac{\langle\frac{\alpha_s}{\pi}FF\rangle}{M^2} + \frac{1}{2}\frac{m_u\langle\bar{u}u\rangle + m_s\langle\bar{s}s\rangle}{M^2} \\ & - \frac{1}{12}\frac{m_u\langle\bar{u}\sigma_{\mu\nu}F^{\mu\nu}u\rangle + m_s\langle\bar{s}\sigma_{\mu\nu}F^{\mu\nu}s\rangle}{M^4} \\ & \left. +\frac{4\pi}{27}\alpha_s\frac{\langle\bar{u}u\rangle^2 + \langle\bar{s}s\rangle^2}{M^4} - \frac{16\pi}{27}\alpha_s\frac{\langle\bar{u}u\rangle\langle\bar{s}s\rangle}{M^4}\right] \\ = \; & M_2^s(\mu^2) + CM^2, \end{aligned} \tag{11}$$

where

$$L = \ln(\frac{M^2}{\Lambda^2})/\ln(\frac{\mu^2}{\Lambda^2}) \tag{12}$$

and C is a constant. In Eq.(11), we moved the contribution from the continuum to the l.h.s. Note that four-quark condensate terms are slightly different from those previously obtained by Belyaev and Blok in the pion case [5].

For a given μ^2, the l.h.s. of Eq.(11) is a linear function of the Borel mass squared M^2 with $M_2^s(\mu^2)$ as a constant term, whereas the l.h.s. of Eq.(11) is a very complicated function of M^2. We approximate the latter by a linear function in the region of M^2 where QSR is applicable. This region of M^2 is obtained according to the following condition; (i) the order of the contribution from the highest order term of OPE is less than 10 % of the l.h.s., and (ii) contribution from the continuum is less than 50 % of the other. In this way, we calculated $M_2^s(\mu^2)$ for various values of the continuum threshold s_0, and find stable s_0, which has the least influence on the result.

We plotted the r.h.s. of Eq.(11) in Fig.2 for both the pion and the kaon cases. Here the values of the continuum thresholds are $s_0 = 0.8\text{GeV}^2$ for the pion and $s_0 = 1.2\text{GeV}^2$ for the kaon. Fig.2

We thus obtain the results,

$$
\begin{aligned}
M_2^s(\mu^2 = 49\text{GeV}^2) &= 0.39 \pm 0.04 \qquad \text{for the pion,} && (13)\\
M_2^s(\mu^2 = 49\text{GeV}^2) &= 0.41 \pm 0.04 \qquad \text{for the kaon,} && (14)
\end{aligned}
$$

Here, for numerical calculations, we used following values: $\langle \frac{\alpha_s}{\pi} FF \rangle = 1.2 \times 10^{-2}\text{GeV}^4$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ (q: u or d) , $m_q \langle \bar{q}q \rangle = -(0.096\text{GeV})^4$, $m_s \langle \bar{s}s \rangle = -(0.21\text{GeV})^4$, $\langle \bar{s}\sigma_{\mu\nu} F^{\mu\nu} s \rangle = 0.8 \langle \bar{q}\sigma_{\mu\nu} F^{\mu\nu} q \rangle$, $\langle \bar{q}\sigma_{\mu\nu} F^{\mu\nu} q \rangle = 2m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = 0.4\text{Gev}^2$, $\alpha_s \langle \bar{q}q \rangle^2 = 1.8 \times 10^{-4}\text{GeV}^6$, $\Lambda = 150\text{MeV}$, $f_\pi = 93\text{MeV}$ and $f_K = 114\text{MeV}$.

Our result in the pion case can be compared with the following values.

(i) NLO analysis of the Drell-Yan data [7]

$$
2 \int_0^1 \mathrm{d}\xi \; \xi f_{\text{valence}}(\xi, \mu^2 = 49\text{GeV}^2) = 0.40 \pm 0.02. \qquad (15)
$$

(ii) Lattice calculation [8]

$$
M_2^s(\mu^2 = 49\text{GeV}^2) = 0.46 \pm 0.07. \qquad (16)
$$

They are consistent with our result.

Next we discuss about the flavor (quark contents) dependence of structure functions. Though the two values, (13) and (14), are close to each other, it does not imply that their dynamical origins are similar, as we can see below. Since the sum rules for these two mesons are the same in their form, differences are in the input values of decay constants, meson masses and quark mass dependent terms ($m_q \langle \bar{q}q \rangle$ and $m_s \langle \bar{s}s \rangle$).

First to see how the difference in decay constants affect the M_2^s values, we put masses of mesons and quarks to be zero (chiral limit). Then we have 0.38 as the M_2^s value for the pion, and 0.34 as that for the kaon. The fact that in the chiral limit the M_2^s value for the kaon is smaller than that for the pion is easily understood from the sum rule Eq.(11). The difference in these two calculations comes from the values of decay constants, where larger decay constant for the kaon results in smaller M_2^s value than that for the pion.

The roles of meson masses and of quark masses are understood in the following manner. In the pion case, even if we put the pion mass and/or

quark masses to be zero, the result is almost unchanged. It can be seen by comparing the chiral limit result 0.38 with 0.39 of Eq.(13). Therefore the meson/quark masses play a minor role in the calculations for the pion.

However the situation is quite different in the kaon case. The finiteness of the meson and quark masses (the mass effect) lift the value of M_2^s for kaon from 0.34 (chiral limit) up to 0.41 of Eq.(14). Moreover each of the meson mass and the quark mass has much larger effect on the M_2^s value for the kaon.

Since the meson mass is appearing in Eq.(11) as $\exp(m_K^2/M^2)$, larger meson mass results in larger M_2^s. And if we simply neglect the kaon mass, we have 0.26 as M_2^s for the kaon. On the other hand, if we simply neglect terms with quark masses, we have 0.51 as M_2^s for the kaon. Though these two effects are quite large, their directions are opposite to each other. Thus their partial cancellation gives the forementioned mass effect.

In summary, we calculated M_2^s values for the pion and the kaon in QCD sum rules, and obtained similar values of M_2^s for these mesons. The value of M_2^s for the pion is consistent with the lattice calculation and with the experimental analysis.

We then discussed about the flavor (quark contents) dependence of structure functions. Though larger decay constant for the kaon makes M_2^s decrease compared to that for the pion, this effect almost cancels the mass effect. Also, each of the kaon mass and quark masses has a large effect on M_2^s for the kaon, and non-zero values of them are essential.

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Figure Caption

Fig.1
Typical diagrams that contribute to $\Pi'^1(Q^2)$. The wavy lines and the curry lines represent the external tensor fields and the background gluon fields, respectively.

Fig.2
The solid lines indicate the OPE side of QSR at $\mu^2 = 49\mathrm{GeV}^2$. We approximate the solid lines by linear lines in the region of M^2 in between the dotted lines.